Math 2326
First Midterm
Khamsi

Problem 1. **Solve the initial-value problem**

$$\frac{dy}{dt} = ty^2 + 2y^2 + t + 2\ , \quad y(0) = 1.$$

Problem 2. **Find the solution to**

$$\frac{dy}{dt} = -5y + \sin(t)\ , \quad y(0) = 1.$$

Problem 3. **A cup of hot chocolate is initially at 110^oF and is left in a room with ambient temperature of 70^oF. Suppose that after one minute, the chocolate cooled 10^oF. Write down the differential equation satisfied by the temperature of the chocolate. Then solve it. How long does it take the hot chocolate to cool to 75^oF?**

Problem 4. **Use Euler's method with the given step size Δt to approximate the solution to the given initial-value problem over the time interval specified.**

$$\frac{dy}{dt} = e^{-\,y/2}, \quad y(0) = 2, \quad 0 \le t \le 3, \quad \Delta t = 0.5$$

Find the exact solution and determine the error at $t = 3$. Draw the graph of the approximated solution as well as the exact one.

Problem 5. **For the one-parameter family**

$$\frac{dy}{dt} = \cos(y) + \alpha$$

identify the bifurcation values of α. Draw the phase lines for $\alpha = -2, -1, -\frac{1}{2}, \frac{1}{2}, 1, 2$.
Classify the constant solutions (if any) as sink, source, or nodes for $\alpha = -2, -1, -\frac{1}{2}, \frac{1}{2}, 1, 2$.
Draw some solutions for $\alpha = -2, -1, -\frac{1}{2}, \frac{1}{2}, 1, 2$.