

## Math Competition: Review 2

### Khamsi

**Problem 1.** [2002] Let  $k$  be a fixed positive integer. The  $n$ -th derivative of  $\frac{1}{x^k - 1}$  has the form  $\frac{P_n(x)}{(x^k - 1)^{n+1}}$  where  $P_n(x)$  is a polynomial. Find  $P_n(1)$ .

**Problem 2.** [2002] Show that, for all integers  $n > 1$ ,

$$\frac{1}{2ne} < \frac{1}{e} - \left(1 - \frac{1}{n}\right)^n < \frac{1}{ne}.$$

**Problem 3.** [2003] Suppose that  $a, b, c, A, B, C$  are real numbers,  $a \neq 0$  and  $A \neq 0$ , such that

$$|ax^2 + bx + c| \leq |Ax^2 + Bx + C|$$

for all real numbers  $x$ . Show that

$$|b^2 - 4ac| \leq |B^2 - 4AC|.$$

**Problem 4.** [2003] Find the minimum value of

$$|\sin x + \cos x + \tan x + \cot x + \sec x + \csc x|$$

for real numbers  $x$ .